SULIT

POLITEKNIK MALAYSIA

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS & KOMPUTER

PEPERIKSAAN AKHIR

SESI DISEMBER 2016

BA601: ENGINEERING MATHEMATICS 5

TARIKH : 03 APRIL 2017
MASA : 8.30 AM – 10.30 AM (2 JAM)

Kertas ini mengandungi **SEMBILAN (9)** halaman bercetak.

Bahagian A: Struktur (2 soalan)
Bahagian B: Struktur (2 soalan)
Bahagian C: Struktur (2 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SECTION A : 50 MARKS

BAHAGIAN A : 50 MARKAH

INSTRUCTION:

This section consists of **TWO (2)** questions with 25 marks each. Answer **ONE(1)** question from each part, and **ONE(1)** question from either part A/B/C.

ARAHAN :

*Bahagian ini mengandungi **DUA (2)** soalan dengan jumlah 25 markah setiap satu. Jawab **SATU(1)** soalan dari setiap bahagian, dan **SATU(1)** soalan selebihnya dari mana-mana bahagian samada A/B/C.*

QUESTION 1

SOALAN 1

CLO1
C1

a) Find the value of the following functions.
Cari nilai bagi fungsi-fungsi berikut.

i) $\tanh(-1)$ [2 marks]
[*2 markah*]

ii) $\operatorname{sech}\left(\frac{1}{4}\right)$ [2 marks]
[*2 markah*]

CLO1
C2

b) Given that $3\sinh x = 6$, evaluate the value of $\cosh x$.
Diberi bahawa $3\sinh x = 6$, *cari nilai bagi* $\cosh x$.

[5 marks]
[5 *markah*]

CLO1 C3

c) Prove that:
Buktikan bahawa:

i) $\cosh 2x = 2\cosh^2 x - 1$ [4 marks]
[*4 markah*]

ii) $\cosh^2 x = 1 + \sinh^2 x$ [5 marks]
[*5 markah*]

CLO1 C3

d) Complete the table below for equation $y = \sinh\frac{x}{2}$. Then sketch the graph in the range given as $-3 \le x < 4$.
Lengkapkan jadual di bawah bagi persamaan $y = \sinh\frac{x}{2}$. *Seterusnya lakarkan graf pada julat* $-3 \le x < 4$. [7 marks]
[*7 markah*]

QUESTION 2
SOALAN 2

CLO1 C1

a) Find the value of the following functions by using the definition of hyperbolic and inverse hyperbolic functions.
Cari nilai bagi fungsi-fungsi yang berikut dengan menggunakan definisi fungsi hiperbolik dan songsangan hiperbolik.

i) $\tanh\sqrt{49}$ [2 marks]
[*2 markah*]

ii) $\cosh^{-1}(\ln 4)$ [2 marks]
[*2 markah*]

iii) $5\text{cosech}^{-1}(-3.5) + \sinh^{-1} e^1$ [3 marks]
[*3 markah*]

CLO1 C2

b) Find the value of y if $2y\sinh x = \tanh x$. Given $x = 2$.
Dapatkan nilai y *jika* $2y\sinh x = \tanh x$. *Diberi* $x = 2$. [3 marks]
[*3 markah*]

CLO1 C2

c) Show that $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$.
Tunjukkan bahawa $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$. [8 marks]
[*8 markah*]

CLO1 C3

d) Find the principle value and sketch a quadrant graph for the following functions:
Dapatkan nilai principal dan lakarkan graf sukuan bagi fungsi-fungsi berikut:

i) $\tan^{-1}\sqrt{7}$ [3 marks]
[*3 markah*]

ii) $5\sin\alpha + 2 = -2$ [4 marks]
[*4 markah*]

SECTION B : 50 MARKS

BAHAGIAN B : 50 MARKAH

QUESTION 3

SOALAN 3

CLO2
C3

a) Differentiate each of the following equations with respect to x:

Bezakan setiap persamaan berikut terhadap x:

i) $y = 5\ln(\cosh 4x)$

[4 marks]

[*4 markah*]

ii) $y = \cot^{-1}(\sinh 4x^3)$

[4 marks]

[*4 markah*]

iii) $y = e^{2x}\sinh^{-1} x$

[5 marks]

[*5 markah*]

iv) $y = \tanh^{-1}\left(\frac{2x}{3}\right)$

[5 marks]

[*5 markah*]

CLO2
C3

b) Given $5y^2 + \tanh 2y = x^3$.

Diberi $5y^2 + \tanh 2y = x^3$.

i) Find $\frac{dy}{dx}$ by using implicit differentiation.

Dapatkan $\frac{dy}{dx}$ *dengan menggunakan pembezaan fungsi tersirat.*

[5 marks]

[*5 markah*]

ii) Evaluate $\frac{dy}{dx}$ when $x = 1$ and $y = 2$.

Nilaikan $\frac{dy}{dx}$ *apabila* $x = 1$ *and* $y = 2$.

[2 marks]

[*2 markah*]

QUESTION 4

SOALAN 4

CLO2 C3

a) Integrate the following functions with the respect of x.

Kamirkan setiap fungsi berikut terhadap x.

i) $\int 4\cosh(2x+3)$ [3 marks]

[*3 markah*]

ii) $\int \frac{e^x}{4+e^{2x}}$ [5 marks]

[*5 markah*]

iii) $\int \frac{dx}{\sqrt{x^2+8x+12}}$ [5 marks]

[*5 markah*]

CLO2 C3

b) Solve $\int \frac{x+1}{(x-2)(x-1)}dx$ by using **integration by partial fractions**.

*Selesaikan $\int \frac{x+1}{(x-2)(x-1)}dx$ dengan menggunakan **kamiran pecahan separa**.*

[6 marks]

[*6 markah*]

CLO2 C3

c) Solve $\int 4xe^{2x}dx$ by using **integration by parts**.

*Selesaikan $\int 4xe^{2x}dx$ dengan menggunakan **kamiran bahagian demi bahagian**.*

[6 marks]

[*6 markah*]

SECTION C : 50 MARKS

BAHAGIAN C : 50 MARKAH

QUESTION 5

SOALAN 5

CLO3 C2

a) Form a differential equation for $y = Ax^3 - Bx + 5$.

Bentukkan persamaan pembezaan bagi $y = Ax^3 - Bx + 5$.

[9 marks]

[*9 markah*]

CLO3 C3

b) Determine the general solution of the following differential equations.

Tentukan penyelesaian am bagi persamaan pembezaan yang berikut.

i) $e^{9x}\frac{dy}{dx} = e^{2x}$

[3 marks]

[*3 markah*]

ii) $2\frac{dy}{dx} = 3x^5$

[3 marks]

[*3 markah*]

iii) $\frac{1}{2x^3}\frac{dy}{dx} = y$

[4 marks]

[*4 markah*]

iv) $\frac{dy}{dx} + \frac{y}{x} = x^3 + 4x - 6$

[6 marks]

[*6 markah*]

QUESTION 6

SOALAN 6

CLO3 C3

a) Solve the following differential equations:
Selesaikan persamaan pembezaan berikut:

i) $(xy+6x)\frac{dy}{dx}-7y=0$ [6 marks]

[*6 markah*]

ii) $\frac{dy}{dx}-5y=e^{5x}\text{cosech}^{2}x$ [7 marks]

[*7 markah*]

CLO3 C2

b) Find the general solution of the second-order differential equations given.
Dapatkan penyelesaian am bagi persamaan pembezaan peringkat kedua berikut.

i) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}=10y$ [5 marks]

[*5 markah*]

ii) $2\frac{d^2y}{dx^2}+3\frac{dy}{dx}+7y=0$ [7 marks]

[*7 markah*]

SOALAN TAMAT

FORMULA ENGINEERING MATHEMATICS 5

HYPERBOLIC FUNCTIONS	INVERSE HYPERBOLIC FUNCTIONS
$\sinh x = \frac{e^x-e^{-x}}{2}$	$\sinh^{-1}x = \ln\left(x+\sqrt{x^2+1}\right); -\infty<x<\infty$
$\cosh x = \frac{e^x+e^{-x}}{2}$	$\cosh^{-1}x = \ln\left(x+\sqrt{x^2-1}\right); x\geq 1$
$\tanh x = \frac{e^x-e^{-x}}{e^x+e^{-x}}$	$\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right); \lvert x\rvert<1$
$coth\ x = \frac{e^x+e^{-x}}{e^x-e^{-x}}; x\neq 0$	$\coth^{-1}x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right); \lvert x\rvert>1$
$\sec h\ x = \frac{2}{e^x+e^{-x}}$	$\text{sech}^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right); 0<x\leq 1$
$\text{cos}\,ech\ x = \frac{2}{e^x-e^{-x}}; x\neq 0$	$\text{cosech}^{-1} = \ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right); x\neq 0$

RECIPROCAL TRIGONOMETRIC IDENTITIES	RECIPROCAL HYPERBOLIC IDENTITIES
$\text{cosec}\ x = \frac{1}{\sin x}$	$\text{cosec}\,h\ x = \frac{1}{\sinh x}$
$\sec x = \frac{1}{\cos x}$	$\sec h\ x = \frac{1}{\cosh x}$
$\cot x = \frac{1}{\tan x}$	$\coth x = \frac{1}{\tanh x}$

TRIGONOMETRIC IDENTITIES	HYPERBOLIC IDENTITIES
$\cos^2 x+\sin^2 x = 1$	$\cosh^2 x-\sinh^2 x = 1$
$1+\tan^2 x = \sec^2 x$	$1-\tanh^2 x = \text{sech}^2 x$
$\cot^2 x+1 = \text{cosec}^2 x$	$\coth^2 x-1 = \text{cosech}^2 x$
$\sin 2x = 2\sin x\cos x$	$\sinh 2x = 2\sinh x\cosh x$
$\cos 2x = \cos^2 x-\sin^2 x$	$\cosh 2x = \cosh^2 x+\sinh^2 x$
$= 2\cos^2 x-1$	$= 2\cosh^2 x-1$
$= 1-2\sin^2 x$	$= 1+2\sinh^2 x$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$
$\sin(x\pm y) = \sin x\cos y\pm\cos x\sin y$	$\sinh(x\pm y) = \sinh x\cosh y\pm\cosh x\sinh y$
$\cos(x\pm y) = \cos x\cos y\mp\sin x\sin y$	$\cosh(x\pm y) = \cosh x\cosh y\pm\sinh x\sinh y$
$\tan(x\pm y) = \frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$	$\tanh(x\pm y) = \frac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$

BASIC OF DIFFERENTIATION

$$\frac{d}{dx}(k) = 0;\ k = \text{constant}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}$$

$$\frac{d}{dx}(\ln|u|) = \frac{1}{u}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u\cdot\frac{du}{dx}$$

BASIC OF INTEGRATION

$$\int k\,du = ku + C\ ;\ k = \text{constant}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C\ ;\ n \neq -1$$

$$\int \frac{1}{u}\,du = \frac{\ln|u|}{\left(\frac{du}{dx}\right)} + C$$

$$\int e^u\,du = \frac{e^u}{\left(\frac{du}{dx}\right)} + C$$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}(\cos u) = -\sin u.\frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u.\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u.\frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u.\frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u.\tan u.\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u.\cot u.\frac{du}{dx}$$

INTEGRATION OF TRIGONOMETRIC FUNCTIONS

$$\int \sin u\,du = \frac{-\cos u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \cos u\,du = \frac{\sin u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \sec^2 u\,du = \frac{\tan u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \operatorname{cosec}^2 u\,du = \frac{-\cot u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \sec u\tan u\,du = \frac{\sec u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \operatorname{cosec} u\cot u\,du = \frac{-\operatorname{cosec} u}{\left(\frac{du}{dx}\right)} + C$$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS

$$\frac{d}{dx}(\cosh u) = \sinh u.\frac{du}{dx}$$

$$\frac{d}{dx}(\sinh u) = \cosh u.\frac{du}{dx}$$

$$\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u.\frac{du}{dx}$$

$$\frac{d}{dx}(\coth u) = -\operatorname{cosech}^2 u.\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{cosech} u) = -\operatorname{cosech} u.\coth u.\frac{du}{dx}$$

INTEGRATION OF HYPERBOLIC FUNCTIONS

$$\int \sinh u\,du = \frac{\cosh u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \cosh u\,du = \frac{\sinh u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \operatorname{sech}^2 u\,du = \frac{\tanh u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \operatorname{cosech}^2 u\,du = \frac{-\coth u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \operatorname{sech} u\tanh u\,du = \frac{-\operatorname{sech} u}{\left(\frac{du}{dx}\right)} + C$$

$$\int \operatorname{cosech} u\coth u\,du = \frac{-\operatorname{cosech} u}{\left(\frac{du}{dx}\right)} + C$$

DIFFERENTIATION OF INVERSE TRYGONOMETRIC FUNCTIONS

$$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx},\ |u|<1$$

$$\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx},\ |u|<1$$

$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\cot^{-1} u) = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx},\ |u|>1$$

$$\frac{d}{dx}(\operatorname{cosec}^{-1} u) = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx},\ |u|>1$$

INTEGRATION OF INVERSE TRYGONOMETRIC FUNCTION

$$\int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\frac{u}{a} + C,\ |u|<a$$

$$\int -\frac{1}{\sqrt{a^2-u^2}}du = \cos^{-1}\frac{u}{a} + C,\ |u|<a$$

$$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int -\frac{1}{a^2+u^2}du = \frac{1}{a}\cot^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{|u|\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C,\ |u|>a$$

$$\int -\frac{1}{|u|\sqrt{u^2-a^2}}du = \frac{1}{a}\operatorname{cosec}^{-1}\frac{u}{a} + C,\ |u|>a$$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS

$$\frac{d}{dx}(\sinh^{-1} u) = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$$

$$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx},\ |u|>1$$

$$\frac{d}{dx}(\tanh^{-1} u) = \frac{1}{1-u^2}\frac{du}{dx},\ |u|<1$$

$$\frac{d}{dx}(\coth^{-1} u) = \frac{1}{1-u^2}\frac{du}{dx},\ |u|>1$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} u) = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx},\ 0<u<1$$

$$\frac{d}{dx}(\operatorname{cosech}^{-1} u) = -\frac{1}{|u|\sqrt{1+u^2}}\frac{du}{dx},\ u\neq 0$$

INTEGRATION OF INVERSE HYPERBOLIC FUNCTIONS

$$\int \frac{1}{\sqrt{a^2+u^2}}du = \sinh^{-1}\frac{u}{a} + C,\ a>0$$

$$\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}\frac{u}{a} + C,\ u>a$$

$$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C;\ |u|<a$$

$$\int \frac{1}{u^2-a^2}du = \frac{1}{a}\coth^{-1}\frac{u}{a} + C;\ |u|>a$$

$$\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}\operatorname{sech}^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{a^2+u^2}}du = -\frac{1}{a}\operatorname{cosech}^{-1}\frac{u}{a} + C$$

INTERGRALS INVOLVING QUADRATIC EXPRESSION

Completing the square

$$ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

FORMULA ENGINEERING MATHEMATICS 5

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION

Homogeneous Equations

- Substitution

$y = vx$ and $\frac{dy}{dx} = v + x\frac{dy}{dx}$

Linear Factors (Integrating Factors)

$$y \bullet IF = \int Q \bullet IF \, dx$$

Where $IF = e^{\int P dx}$

Logarithmic

$a = e^{\ln a}$

$a^x = e^{x \ln a}$

$\int a^x dx = \frac{a^x}{\ln a} + c$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$

1. Real & different roots: $y = Ae^{m_1 x} + Be^{m_2 x}$
2. Real & equal roots: $y = e^{mx}(A + Bx)$
3. Complex roots: $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$