

SULIT



**BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN MALAYSIA**

JABATAN KEJURUTERAAN MEKANIKAL

**PEPERIKSAAN AKHIR
SESI 1 2018/2019**

**BBM3033 : PROBABILITY & STATISTICS FOR ENGINEERING
TECHNOLOGY**

**TARIKH : 07 JANUARI 2019
MASA : 9.00 PAGI – 12.00 PETANG**

Kertas ini mengandungi **SEBELAS (11)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Kertas Graf, Formula dan Jadual Statistik

JANGAN BUKA KERTAS SOALANINI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

Answer ALL questions.

ARAHAN:

Jawab SEMUA soalan.

QUESTION 1**SOALAN 1**

- CLO1 C1 a) In a study, a life span of 100 halogen lamps manufactured by a factory is recorded in **Table 1a**.

Dalam satu kajian, jangka hayat 100 lampu halogen yang dibuat oleh sebuah kilang direkodkan dalam Jadual 1a.

Table 1a / Jadual 1a

| Life span (days) | Frequency |
|------------------|-----------|
| 1 – 10 | 12 |
| 11 – 20 | 16 |
| 21 – 30 | 24 |
| 31 – 40 | 27 |
| 41 – 50 | 11 |
| 51 – 60 | 10 |

- (i) State the mode class and the median class.

Nyatakan kelas mod dan kelas median.

[2 marks]

[2 markah]

- (ii) Find the class boundary, midpoint and cumulative frequency for the data.

Cari sempadan kelas, titik tengah dan kekerapan kumulatif bagi data tersebut.

[3 marks]

[3 markah]

CLO1
C3

- b) Table 1b shows the information on the amounts of electric bills (RM) for a sample of 50 families during August 2018.

Jadual 1b menunjukkan maklumat jumlah bil elektrik (RM) untuk sampel 50 buah keluarga sepanjang bulan Ogos 2018.

Table 1b / Jadual 1b

| Amount of bills (RM) | Number of families |
|----------------------|--------------------|
| 1 – 20 | 5 |
| 21 – 40 | 16 |
| 41 – 60 | 11 |
| 61 – 80 | 10 |
| 81 – 100 | 8 |

Calculate the mean and mode.

Kirakan min dan mod.

[5 marks]

[5 markah]

CLO2
C4

- c) Table 1c shows the frequency distribution table of the time spent in a week by 50 students in preparing for their final examination.

Jadual 1c menunjukkan jadual taburan kekerapan bagi masa yang diperuntukkan dalam seminggu oleh 50 orang pelajar dalam membuat persediaan bagi peperiksaan akhir mereka.

Table 1c/ Jadual 1c

| Time (hours) | Number of students |
|--------------|--------------------|
| 5 – 9 | 2 |
| 10 – 14 | 4 |
| 15 – 19 | 7 |
| 20 – 24 | 9 |
| 25 – 29 | 12 |
| 30 – 34 | 11 |
| 34 – 39 | 4 |
| 40 – 44 | 1 |

- (i) Construct the cumulative frequency table. Hence, draw the ogive.

Bina jadual kekerapan kumulatif. Seterusnya, bina ogif.

[7 marks]

[7 markah]

- (ii) Based on the ogive in (i), find:

Berdasarkan ogif dalam (i), cari:

- a. the median and interquartile range for the time spent in a week by 50 students in preparing for their final examination,

median dan julat antara kuartil bagi masa yang diperuntukkan dalam seminggu oleh 50 orang pelajar dalam membuat persediaan bagi peperiksaan akhir mereka,

[4 marks]

[4 markah]

- b. the value of x if 40% of the students spent less than x hours in a week preparing for their final examination,

nilai x jika 40% pelajar memperuntukkan kurang daripada x jam dalam seminggu untuk membuat persediaan bagi peperiksaan akhir mereka,

[2 marks]

[2 markah]

- c. the percentage number of students who spent more than 36 hours in a week preparing for their final examination.

peratusan bilangan pelajar yang memperuntukkan masa melebihi 36 jam dalam seminggu untuk membuat persediaan bagi peperiksaan akhir mereka.

[2 marks]

[2 markah]

QUESTION 2**SOALAN 2**

CLO1

C2

- a) Given that the letters of the word ‘PROBABILITY’ is jumbled up and then is arranged in a row.

Diberi huruf-huruf dalam perkataan ‘PROBABILITY’ dicampur adukkan dan kemudian disusun dalam satu baris.

- (i) Identify the number of arrangements can be formed in a line?

Kenal pasti bilangan susunan boleh dibentuk dalam satu baris?

[2 marks]

[2 markah]

- (ii) Identify the number of arrangements can be formed if the first letter is “O”?

Kenal pasti bilangan susunan boleh dibentuk dalam satu baris jika huruf yang pertama adalah “O”?

[3 marks]

[3 markah]

CLO2

C2

- b) A survey reports that 10% of Malaysia citizens own computers. 200 people are randomly selected to identify whether they own a computer or not.

Satu kajian melaporkan bahawa 10% rakyat Malaysia memiliki komputer. 200 orang telah dipilih secara rawak bagi mengenal pasti sama ada mereka memiliki komputer atau tidak.

- (i) Calculate the mean, variance and standard deviation of the people who owns a computer?

Kirakan min, varians dan sisihan piawai bagi orang yang memiliki komputer?

[5 marks]

[5 markah]

CLO2
C4

- (ii) Calculate the probability that 10 or more people own a computer.

Hitungkan kebarangkalian 10 atau lebih orang memiliki komputer.

[5 marks]

[5 markah]

- c) The Malaysia Automobile Association reports that the average time taken to respond to an emergency call is 25 minutes. Assume the variable is approximately normal distributed and the standard deviation of the population is 4.5 minutes. Find the probability of how many calls will be responded:

Persatuan Automobil Malaysia melaporkan bahawa purata masa yang diambil untuk menjawab satu panggilan kecemasan adalah 25 minit. Andaikan pemboleh ubah adalah dianggarkan tertabur secara normal dan sisihan piawai bagi populasi adalah 4.5 minit. Cari kebarangkalian bilangan panggilan akan dijawab:

- (i) between 15 and 20 minutes
di antara 15 minit hingga 20 minit

[7 marks]

[7 markah]

- (ii) less than 12 minutes
kurang daripada 12 minit

[3 marks]

[3 markah]

QUESTION 3**SOALAN 3**CLO1
C2

- a) A sample of 12 adults which randomly selected were asked how much (in RM) they usually spend on books for general reading per year. Assume that such expenses for all adults who buy books for general reading have an approximately normal distributed.

Sampel 12 orang dewasa yang dipilih secara rawak telah ditanya berapa banyak (dalam RM) yang mereka biasa peruntukkan untuk membeli buku untuk bacaan umum dalam setahun. Anggapkan bahawa perbelanjaan untuk semua orang dewasa yang membeli buku untuk bacaan umum adalah dianggarkan tertabur secara normal.

| | | | |
|--------|--------|--------|--------|
| 540.30 | 420.50 | 723.00 | 145.20 |
| 925.40 | 101.10 | 239.60 | 109.70 |

- (i) Find the best point estimate for the population mean.

Cari titik anggaran terbaik bagi min populasi.

[3 marks]

[3 markah]

- (ii) Hence, construct a 98% confidence interval for the population mean.

Seterusnya, bina selang keyakinan 98% bagi min populasi.

[10 marks]

[10 markah]

- (iii) Calculate the maximum error of estimation for part (ii).

Kira ralat maksimum bagi penganggaran bagi bahagian (ii).

[2 marks]

[2 markah]

CLO1
C3

- b) A random sample of 51 airline passengers at the Kuala Lumpur International Airport (KLIA) shows the mean times spent for waiting in line to check in at the tickets counter was 31 minutes with the standard deviation of 7 minutes. Assume that such waiting times for all passengers are normally distributed.

Sampel rawak daripada 51 orang penumpang syarikat penerbangan di Lapangan Terbang Antarabangsa Kuala Lumpur (KLIA) menunjukkan bahawa min masa yang di ambil untuk menunggu di barisan untuk daftar masuk di kaunter tiket adalah 31 minit dengan sisihan piawai 7 minit. Anggapkan bahawa waktu menunggu bagi semua penumpang tertabur secara normal.

- (i) Construct a 99% confidence interval for the mean times spend for waiting in line by all passengers in the airport.

Bina selang keyakinan 99% untuk min masa yang di ambil untuk menunggu di barisan bagi semua penumpang di lapangan terbang tersebut.

[4 marks]

[4 markah]

- (ii) Construct a 99% confidence interval for the standard deviation time spend for waiting in line by all passengers in the airport.

Bina selang keyakinan 99% untuk sisihan piawai masa yang di ambil untuk menunggu di barisan bagi semua penumpang di lapangan terbang tersebut.

[6 marks]

[6 markah]

QUESTION 4**SOALAN 4**

CLO1

C1

- a) Using the z-table, find the critical values for $\alpha = 0.05$ (two tailed test) and $\alpha = 0.01$ (Left-tailed test), then draw a diagram to indicate both significance values.

Dengan menggunakan jadual z, cari nilai kritikal bagi $\alpha = 0.05$ (ujian dua-hujung) dan $\alpha = 0.01$ (ujian satu-hujung, kiri), kemudian lukis gambar rajah untuk menunjukkan kedua-dua nilai keertian.

[5 marks]

[5 markah]

CLO2

C3

- b) The **Table 4a** shows the observed pollution indexes of air samples in two areas of a city. Test the hypothesis that the mean pollution indexes are the same for the two areas. (Use $\alpha = 0.05$)

Jadual 4a menunjukkan indeks pencemaran yang dicerap dalam sampel udara di dua kawasan sebuah bandar. Uji hipotesis bahawa min indeks pencemaran adalah sama untuk kedua-dua kawasan. (Gunakan $\alpha = 0.05$)

[10 marks]

[10 markah]

Table 4a/ Jadual 4a

| Area A/ Kawasan A | | Area B/ Kawasan B | |
|-------------------|------|-------------------|------|
| 2.92 | 4.69 | 1.84 | 3.44 |
| 1.88 | 4.86 | 0.95 | 3.69 |
| 5.35 | 5.81 | 4.26 | 4.95 |
| 3.81 | 5.55 | 3.18 | 4.47 |

CLO2
CS

- c) The average systolic blood pressure of a normal male is normally to be about 129. Measurements of systolic blood pressure on a sample of 12 adult males from a community whose dietary habits are suspected of causing high blood pressure are listed below: (Use: $\alpha = 0.01$)

Min tekanan darah sistolik bagi lelaki normal kebiasaannya sekitar 129. Pengukuran tekanan darah sistolik ke atas sampel 12 orang dewasa lelaki dari komuniti tabiat pemakanan disyaki menyebabkan tekanan darah tinggi disenaraikan di bawah: (Gunakan: $\alpha = 0.01$)

115 134 131 143 130 154 119 137 155 130 110 138

Table 4b/ Jadual 4b

| T-Test | | | | | |
|-----------------------|------------------|---------|-----------------|-----------------|---|
| One-Sample Statistics | | | | | |
| | N | Mean | Std. Deviation | Std. Error Mean | |
| Blood Pressure | 12 | 133.000 | 13.04144 | 4.0245 | |
| One-Sample Test | | | | | |
| | Test Value = 129 | | | | |
| | t | df | Sig. (2-tailed) | Mean Difference | 99% Confidence Interval of the Difference |
| Blood Pressure | -9.439 | 11 | .3416 | 4.0000 | Lower Upper |
| | | | | | -6.9390 14.9390 |

- (i) State the null and alternative hypotheses that the average systolic blood pressure of the community is more than 129?

Nyatakan hipotesis nol dan hipotesis alternatif bahawa min tekanan darah sistolik bagi komuniti adalah lebih daripada 129?

[2 marks]

[2 markah]

- (ii) Based on the SPSS output on Table 4b, prove that the test statistic is $t = 0.9939$ and state your conclusion based on the significance value?

Berdasarkan hasil data SPSS dalam Jadual 4b, buktikan ujian statistik adalah $t = 0.9939$ dan nyatakan kesimpulan anda berdasarkan nilai signifikan.

[8 marks]

[8 markah]

SOALAN TAMAT

FORMULA SHEET FOR PROBABILITY & STATISTICS FOR ENGINEERING TECHNOLOGY

| <u>DESCRIPTIVE STATISTICS</u> | |
|---|---|
| <u>Ungrouped Data</u> | <u>Grouped Data</u> |
| Mean, $\bar{X} = \frac{\sum X}{n}$ Population Variance, $\sigma^2 = \frac{\sum (X - \mu)^2}{N}$ Sample Variance, $s^2 = \frac{\sum (X - \bar{X})^2}{n-1}$ Or $s^2 = \frac{\sum X^2 - \left[\frac{(\sum X)^2}{n} \right]}{n-1}$ Population Standard Deviation, $\sigma = \sqrt{\sigma^2}$ Sample Standard Deviation, $s = \sqrt{s^2}$ | Mean, $\bar{X} = \frac{\sum f \cdot X_m}{n}$ Population Variance, $\sigma^2 = \frac{\sum f(X - \mu)^2}{N}$ Sample Variance, $s^2 = \frac{\sum f(X - \bar{X})^2}{n-1}$ Or $s^2 = \frac{\sum f X_m^2 - \left[\frac{(\sum f X_m)^2}{n} \right]}{n-1}$ Median, $M = L_M + \left(\frac{\frac{N}{2} - F}{f_M} \right) C$ Mode, $M_o = L_{M_o} + \left(\frac{d_1}{d_1 + d_2} \right) C$ |
| <u>PROBABILITY & STATISTICS</u> | |
| Addition Rule (mutually exclusive events), $P(A \cup B) = P(A) + P(B)$ Addition Rule (events not mutually exclusive), $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Multiplication Rule (Independent event), $P(A \cap B) = P(A) \cdot P(B)$ Multiplication Rule (dependent event), $P(A \cap B) = P(A)P(B A)$ | Conditional Probability, $P(B A) = \frac{P(A \cap B)}{P(A)}$ Conditional Probability, $P(B A) = \frac{P(A \cap B)}{P(A)}$ Complementary events, $P(\bar{A}) = 1 - P(A)$ Permutation Rule, ${}_n P_r = \frac{n!}{(n-r)!}$ Combination Rule, ${}_n C_r = \frac{n!}{(n-r)! r!}$ |

| | |
|---|---|
| <p>Mean for a probability distribution, $\mu = \sum [X \cdot P(X)]$</p> <p>Variance and standard deviation for a probability distribution. $\sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$</p> <p>$\sigma = \sqrt{\sum [X^2 \cdot P(X)] - \mu^2}$</p> <p>Expectation, $E(X) = \sum [X \cdot P(X)]$</p> <p>Binomial probability, $P(X) = \frac{n!}{(n-X)!X!} \cdot p^x \cdot q^{n-x}$</p> <p>Mean for binomial distribution, $\mu = np$</p> <p>Variance and standard deviation for the binomial distribution, $\sigma^2 = npq$ $\sigma = \sqrt{npq}$</p> | <p>Normal distribution</p> <p>Standard score, $z = \frac{X - \mu}{\sigma}$ or $\frac{X - \bar{X}}{s}$</p> <p>Mean of sample mean, $\mu_{\bar{X}} = \mu$</p> <p>Standard error of the means, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$</p> <p>Central limit theorem formula, $z = \frac{X - \mu}{\sigma/\sqrt{n}}$</p> |
|---|---|

SAMPLING AND ESTIMATION

| | |
|---|--|
| <p>z confidence interval for means, $\bar{X} - z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right) < \mu < \bar{X} + z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right)$</p> <p>$t$ confidence interval for means, $\bar{X} - t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$</p> <p>Sample size for means, $n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$, where E is the maximum error of estimate.</p> | <p>Confidence interval for a proportion, $\hat{p} - (z_{\alpha/2}) \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\alpha/2}) \sqrt{\frac{\hat{p}\hat{q}}{n}}$</p> <p>Sample size for proportion, $n = \hat{p}\hat{q} \left(\frac{z_{\alpha/2}}{E} \right)^2$ where E is the maximum error of estimate.</p> <p>Confidence interval for variance, $\frac{(n-1)s^2}{\chi^2_{right}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{left}}$</p> <p>Confidence interval for standard deviation, $\sqrt{\frac{(n-1)s^2}{\chi^2_{right}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{left}}}$</p> |
|---|--|

HYPOTHESIS TESTING

Test for the population mean

$$z \text{ test, } z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}, \text{ variance known}$$

$$z \text{ test, } z = \frac{\bar{X} - \mu}{s / \sqrt{n}}, \text{ variance unknown}$$

$$t \text{ test, } z = \frac{\bar{X} - \mu}{s / \sqrt{n}}, \text{ small sample}$$

Test for two population mean

Variances known:

$$\text{Test statistics, } z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \text{ for}$$

Variances unknown for large samples:

$$\text{Test statistics, } z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \text{ for}$$

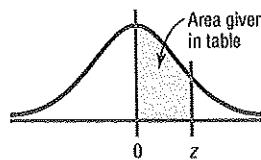
Variances unknown for small samples:

$$\text{Test statistics, } t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \text{ for small}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Table B-1 The Standard Normal Distribution

| <i>z</i> | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |

For *z* values greater than 3.49, use 0.4999.

| | | The <i>t</i> Distribution | | | | |
|----------------------|---------------------|---------------------------|--------------------|--------|--------------------|--------------------|
| Confidence intervals | | 80% | 90% | 95% | 98% | 99% |
| d.f. | One tail, α | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| | Two tails, α | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 |
| 1 | | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 32 | | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 |
| 34 | | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 |
| 36 | | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 |
| 38 | | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 |
| 40 | | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 45 | | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 |
| 50 | | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 |
| 55 | | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 |
| 60 | | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 65 | | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 |
| 70 | | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 75 | | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 |
| 80 | | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 90 | | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 |
| 100 | | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 |
| 500 | | 1.283 | 1.648 | 1.965 | 2.334 | 2.586 |
| 1000 | | 1.282 | 1.646 | 1.962 | 2.330 | 2.581 |
| (z) ∞ | | 1.282 ^a | 1.645 ^b | 1.960 | 2.326 ^c | 2.576 ^d |

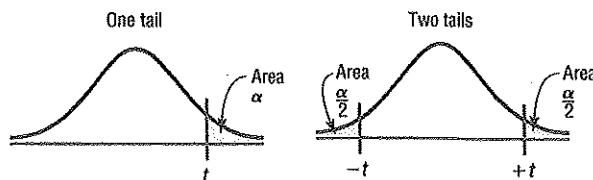
^aThis value has been rounded to 1.28 in the textbook.^bThis value has been rounded to 1.65 in the textbook.^cThis value has been rounded to 2.33 in the textbook.^dThis value has been rounded to 2.58 in the textbook.Source: Adapted from W. H. Beyer, *Handbook of Tables for Probability and Statistics*, 2nd ed., CRC Press, Boca Raton, Fla., 1986. Reprinted with permission.

Table 6 The Chi-Square Distribution

| Degrees of freedom | α | | | | | | | | | |
|--------------------|----------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | — | — | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.071 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.299 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.262 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.194 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.257 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.954 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

Source: Donald B. Owen, *Handbook of Statistics Tables*, The Chi-Square Distribution Table, © 1962 by Addison-Wesley Publishing Company, Inc. Copyright renewal © 1990. Reprinted by permission of Pearson Education, Inc.

