SULIT

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI 2 : 2016/2017

BBM3013 : ADVANCED CALCULUS FOR ENGINEERING TECHNOLOGY

TARIKH : 05 JUN 2017
MASA : 9.00 PAGI – 12.00 TGH (3 JAM)

Kertas ini mengandungi **ENAM (6)** halaman bercetak.

Subjektif (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:
This section consists of **FOUR (4)** structured questions. Answer **ALL** questions.

ARAHAN:
*Bahagian ini mengandungi **EMPAT (4)** soalan berstruktur. Jawab **SEMUA** soalan.*

QUESTION 1
SOALAN 1

CLO 1
C1

(a) Form the differential equations for the given general solutions.
Bentukkan persamaan pembezaan bagi penyelesaian yang diberi.

i. $y = Ax + x^2$ [5 marks]

[5 *markah*]

ii. $y = x^2 + \frac{A}{x}$ [5 marks]

[5 *markah*]

CLO 1
C2

(b) Find the general solutions of the following differential equations.
Cari penyelesaian umum bagi persamaan pembezaan yang berikut.

i. $y' = \frac{e^{-x} + e^{2x} + e^{3x}}{e^{x}}$ [4 marks]

[4 *markah*]

ii. $x\frac{dy}{dx} = 4y + x^5$ [6 marks]

[6 *markah*]

CLO 2
C4

(c) Solve the differential equation, $x\frac{dy}{dx} - 3 = 2(y + \frac{dy}{dx})$ which satisfies the condition $y = 0$ when $x = 3$. [10 marks]

Selesaikan persamaan pembezaan $x\frac{dy}{dx} - 3 = 2(y + \frac{dy}{dx})$ *yang memenuhi syarat* $y=0, x=3$. [10 *markah*]

QUESTION 2
SOALAN 2

CLO1
C2

(a) Solve the following equations:
Selesaikan persamaan di bawah:

i. $y''-y'-6y=0$ [3 marks]
[3 *markah*]

ii. $4\frac{d^2y}{dx^2}+12\frac{dy}{dx}+9y=0$ [3 marks]
[3 *markah*]

CLO2
C3

(b) Given a differential equation,

$$y''-3y'+2y=1.$$

Diberi satu persamaan pembezaan,

$$y''-3y'+2y=1.$$

i. Find the complementary solution, y_c of the given differential equation.
Cari penyelesaian komplementari, y_c bagi persamaan pembezaan yang diberi.
[3 marks]
[3 *markah*]

ii. Find the Wronskian determinant.
Cari penentu Wronskian.
[4 marks]
[4 *markah*]

iii. Find the general solution for the given differential equation.
Cari penyelesaian umum bagi persamaan pembezaan yang diberi.
[7 marks]
[7 *markah*]

QUESTION 3
SOALAN 3

CLO1
C2

(a) By using the Canonical form, classify each of the following equation as elliptic, hyperbolic or parabolic.
Dengan menggunakan bentuk Kanonikal, tentukan sama ada persamaan berikut adalah elliptik, hiperbolik atau parabolik.

i. $\frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x \partial y} - 3\frac{\partial^2 f}{\partial y^2} = 0$ [2marks]
[*2 markah*]

ii. $\frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} = 0$ [2 marks]
[*2 markah*]

iii. $4\frac{\partial^2 f}{\partial x^2} + 4\frac{\partial^2 f}{\partial x \partial y} + 1\frac{\partial^2 f}{\partial y^2} = 0$ [2 marks]
[*2 markah*]

CLO2
C4

(b) Find the general solution for each of the following second order partial differential equation.
Cari penyelesaian umum bagi setiap persamaan separa peringkat kedua yang berikut.

i. $\frac{\partial^2 u}{\partial x \partial y} = 3x - 5y$ [2 marks]
[2 *markah*]

ii. $\frac{\partial^2 f}{\partial u \partial v} = 2u^3v^2 - -3u^2v - 4u$ [3 marks]
[3 *markah*]

iii. $U_{xy} = e^x + \sin y$ [3 marks]
[3 *markah*]

CLO2
C4

(c) Solve each of the following second order partial differential equation. Give your answers in the form of $u(x,y)=F(mx+y)$.

Selesaikan setiap persamaan separa peringkat kedua berikut. Beri jawapan anda dalam bentuk $u(x,y)=F(mx+y)$.

i. $25\dfrac{\partial^2 u}{\partial x^2}-20\dfrac{\partial^2 u}{\partial x\partial y}+4\dfrac{\partial^2 u}{\partial y^2}=0$ [3 marks]

[3 *markah*]

ii. $6\dfrac{\partial^2 u}{\partial x^2}+3\dfrac{\partial^2 u}{\partial x\partial y}-30\dfrac{\partial^2 u}{\partial y^2}=0$ [3 marks]

[3 *markah*]

QUESTION 4
SOALAN 4

CLO1 C1

(a) By using Laplace Transfrom Table, determine the Laplace transform for each of the following function:

Dengan menggunakan Jadual Jelmaan Laplace, tentukan jelmaan Laplace bagi fungsi berikut:

i. $f(t)=5$ [1 mark]
[1 *markah*]

ii. $f(t)=e^{2t}+e^{-3t}$ [2 marks]
[2 *markah*]

iii. $f(t)=t-\frac{t^3}{2}$ [2 marks]
[2 *markah*]

CLO1 C3

(b) Find the following inverse Laplace transforms.

Cari songsangan bagi jelmaan Laplace berikut.

i. $L^{-1}\left\{\frac{20}{s^2+4}\right\}$ [3 marks]
[3 *markah*]

ii. $L^{-1}\left\{\frac{5}{s}-\frac{10}{2s-4}\right\}$ [3 marks]
[3 *markah*]

iii. $L^{-1}\left\{\frac{6}{s^2+4}-\frac{2s}{s^2+9}-\frac{1}{s^4}\right\}$ [4 marks]
[4 *markah*]

CLO2 C5

(c) Use the Laplace transform to solve the given initial value problem.

Dengan menggunakan jelmaan Laplace, selesaikan masalah nilai awal yang diberi.

$$y''-3y'+2y=0;\ y(0)=8,\ y'(0)=11$$ [15 marks]
[15 *markah*]

SOALAN TAMAT

FORMULA

Basic Differentiation	Basic Integration
$\frac{dy}{dx}=\frac{du}{dx}v+u\frac{dv}{dx}$	$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C,\quad n\neq -1$
$\frac{dy}{dx}=\frac{\frac{du}{dx}v-u\frac{dv}{dx}}{v^2}$	$\int u\,dv=uv-\int v\,du$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}\,du=\frac{1}{a}e^{ax}+C$
$\frac{d}{dx}\left(\ln\lvert x\rvert\right)=\frac{1}{x}$	$\int\frac{1}{x}\,dx=\ln\lvert x\rvert+C$
$\frac{d}{dx}[\sin(ax)]=a\cos(ax)$	$\int\sin(ax)\,dx=-\frac{1}{a}\cos(ax)+C$
$\frac{d}{dx}[\cos(ax)]=-a\sin(ax)$	$\int\cos(ax)\,dx=\frac{1}{a}\sin(ax)+C$
$\frac{d}{dx}[\tan(ax)]=a\sec^2(ax)$	$\int\sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+C$

First Order Differential

Separable:

$$\frac{dy}{dx}=f(x)\cdot g(y)$$

Homogeneous:

$P(x,y)dx+Q(x,y)dy=0$, P and Q have same degree.

Exact:

$$P(x,y)dx+Q(x,y)dy=0,\quad \frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$$

Linear:

$$\frac{dy}{dx}+P(x)y=Q(x)$$

$$ye^{\int P(x)\,dx}=\int Q(x)\cdot e^{\int P(x)\,dx}dx+c$$

Bernoulli:

$$\frac{dy}{dx}+P(x)y=Q(x)y^n$$

$$y^{1-n}e^{\int(1-n)P(x)\,dx}=\int(1-n)\cdot Q(x)\cdot e^{\int(1-n)P(x)\,dx}dx$$

Second Order Differential

Quadratic Equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Roots real and different $m=m_1$ and $m=m_2$

$\therefore$ Solution is $y=Ae^{m_1x}+Be^{m_2x}$

Roots real and equal $m_1 = m_2$

$\therefore$ Solution is $y = e^{m_1 x}(A + Bx)$

Complex roots $m = \alpha \pm \beta i$

$\therefore$ Solution is $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$$\frac{d^2y}{dx^2} + n^2 y = 0 \qquad m = \pm n_i$$

$\therefore$ Solution is $y = A\cos nx + B\sin nx$

$$\frac{d^2y}{dx^2} - n^2 y = 0 \qquad m = \pm n$$

$\therefore$ Solution is $y = A\cosh nx + B\sinh nx$

Particular Integral

If $G(x)$	Assume (y_p)
k (constant)	A
kx	$Ax + B$
kx^2	$Ax^2 + Bx + C$
$k\sin\alpha x$ or $k\cos\alpha x$	$A\cos\alpha x + B\sin\alpha x$
$k\sinh\alpha x$ or $k\cosh\alpha x$	$A\cosh\alpha x + B\sinh\alpha x$
e^{kx}	Ae^{kx}

Wronskian Determinant

$$W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= y_1 y_2' - y_2 y_1'$$

Particular solution

$$y_p = -y_1\int \frac{(y_2)(G(x))}{W}dx + -y_2\int \frac{(y_1)(G(x))}{W}dx$$

Second Order Partial Differential
$B^2 - AC > 0$ Hyperbolic equation $B^2 - AC < 0$ Elliptic equation $B^2 - AC = 0$ Parabolic equation

Laplace Transform Table

No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
3.	$t^n,\ n=1,2,3...$	$\frac{n!}{s^{n+1}}$	15.	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$
4.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16.	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at}\sinh\omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at}\sinh\omega t$	$\frac{\omega}{(s+a)^2-\omega^2}$
7.	$t^n.e^{at}\ \ n=1,2,3...$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at}\cosh\omega t$	$\frac{s+a}{(s+a)^2-\omega^2}$
8.	$t^n.f(t)$	$(-1)^n\frac{d^n}{ds^n}[F(s)]$	20.	$f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$
9.	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	21.	$\int_o^t f(u)du$	$\frac{F(s)}{s}$
10.	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
11.	$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s)-y(0)$
12.	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	24.	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s)-sy(0)-y'(0)$