SECTION A

STRUCTURED (25 marks)

INSTRUCTION:

This section consists of **TWO (2)** structured questions. Answer **ONE (1)** question only.

QUESTION 1

a) Expand the expression of

i. $(2-3x)^5$ using the Pascal's Triangle. [CLO1: C3] (5 marks)

ii. $\frac{2x}{(1-2x)^2}$ until the first four terms. [CLO1: C3] (5 marks)

b) Determine the coefficient of x^3 in the expansion of $(2x-\frac{1}{3x^2})^{12}$

[CLO1: C3] (7 marks)

c) Expand the expression of $(1+2x)^5$ using the Binomial Theorem up to x^4.

Hence, find the value of $(1.02)^5$ correct to 3 decimal places.

[CLO1: C3] (8 marks)

POLITEKNIK
Jabatan Pengajian Politeknik

EXAMINATION AND EVALUATION DIVISION
DEPARTMENT OF POLYTECHNIC EDUCATION

(MINISTRY OF HIGHER EDUCATION)

MATHEMATICS, SCIENCE & COMPUTER DEPARTMENT

FINAL EXAMINATION

JUNE 2012 SESSION

BA501: ENGINEERING MATHEMATICS 4

DATE: 17 NOVEMBER 2012 (SATURDAY)
DURATION: 2 HOURS (8.30AM – 10.30 AM)

This paper consists of **NINE (9)** pages including the front page.

Section A: Structured (2 questions – answer ONE (1) question only)
Section B: Structured (2 questions – answer ONE (1) question only)
Section C (JKE, JKP, JKPK): Structured (2 questions – answer ONE (1) question only)
Section D (JKM): Structured (2 questions – answer ONE (1) question only)
Section E: Answer ONE (1) question from section A, B or C (for JKE, JKP and JKPK) and section A, B or D (for JKM).

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(The CLO stated is for reference only.)

QUESTION 2

a) Determine the power series of $(1-2x)\,e^{-2x}$ until the first five terms.

[CLO1:C3] (8 marks)

b) Expand the expression of $\ln\left(\frac{1+3x}{(1-2x)^2}\right)$ until the first four terms.

[CLO1:C3] (10 marks)

c) Find the first four terms of $f(x) = \sin(2-x)$ using the Mac Laurin series.

[CLO1:C3] (7 marks)

SECTION B

STRUCTURED (25 marks)

INSTRUCTION:

This section consists of **TWO (2)** structured questions. Answer **ONE (1)** question only.

QUESTION 3

a) Given the coordinates of A(4 , -2 , 5) , B (1 , 3 , 0) and C (2 , 3, -1) . Find

i. $\overrightarrow{AB}$ [CLO2:C2] (3 marks)

ii. $\overrightarrow{BC}$ [CLO2:C2] (3 marks)

iii. $\overrightarrow{OA} + 2\overrightarrow{OB}$ [CLO2:C2] (2 marks)

iv. $\overrightarrow{OA} \bullet \overrightarrow{OB}$ [CLO2:C2] (3 marks)

v. $\overrightarrow{OB} \bullet \overrightarrow{OC}$ [CLO2:C2] (3 marks)

b) Given the position vectors, $\overrightarrow{OP} = (3 , 4 , 5)$, $\overrightarrow{OQ} = (-2, 3, 1)$ and $\overrightarrow{OR} = (4, 3, -2)$. Determine

i. $\overrightarrow{OP} \times \overrightarrow{OR}$ [CLO2:C3] (3 marks)

ii. $\overrightarrow{OP} \bullet (\overrightarrow{OQ} \times \overrightarrow{OR})$ [CLO2:C3] (5 marks)

iii. $\overrightarrow{OP} \times (\overrightarrow{OQ} \times \overrightarrow{OR})$ [CLO2:C3] (3 marks)

SECTION C (JKE, JKP, JKPK):

STRUCTURED (25 marks)

INSTRUCTION:

This section consists of **TWO (2)** structured questions. Answer **ONE (1)** question only.

QUESTION 5

a) Calculate the Laplace Transform of $f(t)$ using the definition of $F(s) = \int_0^\infty e^{-st} f(t)\,dt$.

i. $f(t) = -9$ [CLO3:C3] (7 marks)

ii. $f(t) = 7e^{4t}$ [CLO3:C3] (7 marks)

b) Determine the Laplace Transform for each the following functions

i. $f(t) = (3t-2)^2$ [CLO3:C3] (3 marks)

ii. $f(t) = \frac{e^{5t}}{2} + 5 - 3t$ [CLO3:C3] (3 marks)

iii. $f(t) = 4\cos 5t + 2\sinh 3t$ [CLO3:C3] (3 marks)

iv. $f(t) = t^2 e^{-3t}$ [CLO3:C3] (2 marks)

QUESTION 4

Convert the fractions below to partial fractions.

a) $\frac{x+1}{x^2+2x-3}$ [CLO2:C3] (6 marks)

b) $\frac{2x^4-8x^2+5x-2}{x^3-4x}$ [CLO2:C3] (9 marks)

c) $\frac{5x^2+3x-2}{x^2(2x^2+1)}$ [CLO2:C3] (10 marks)

QUESTION 6

a) Find the centre, vertices, foci and asymptotes of the hyperbola $y^2 - 4x^2 = 16$. [CLO4:C3] (7 marks)

b) Find the centre, foci, vertices, eccentric and directrixs for the ellipse $\frac{(x-3)^2}{16} + \frac{(y+4)^2}{9} = 1$. Hence, sketch the graph by showing the vertices and centre of the Ellipse.[CLO4:C3]

(18 marks)

SECTION E

Answer **ONE(1)** question from section **A**, **B** or **C** (for **JKE**, **JKP** and **JKPK**) and section **A**, **B** or **D** (for **JKM**).

QUESTION 6

a) Find the Inverse Laplace Transform for the fractions below:

i. $\frac{3}{2s-15}$ [CLO3:C3] (3 marks)

ii. $\frac{6s+4}{s^2+16}$ [CLO3:C3] (4 marks)

b) Determine the Inverse Laplace Transform using the Partial Fraction method:

i. $\frac{4}{s(s-1)}$ [CLO3:C3] (8 marks)

ii. $\frac{s}{(s+1)(s^2+1)}$ [CLO3:C3] (10 marks)

FORMULA BA501 - ENGINEERING MATHEMATICS 4

Binomial Expansion

1	$(a+x)^n = a^n + {}^nC_1 a^{n-1}x + {}^nC_2 a^{n-2}x^2 + \ldots\ldots + x^n$	(n = positive integer)
2	$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \ldots\infty$	(n = negative interger or fraction)

Power Series

1	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots. + \frac{x^n}{n!}$	
2	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots. + (-1)^{n-1}\frac{x^n}{n}$	
3	$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots.. + \frac{f^n(0)x^n}{n!}$	(MACLAURIN)
4	$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)(x-x_0)^3}{3!} + \ldots. + \frac{f^n(x_0)(x-x_0)^n}{n!}$	(TAYLOR)

Vector & Scalar

1	$\overline{A} \bullet \overline{B} = a_1a_2 + b_1b_2 + c_1c_2$	3	$\cos\theta = \frac{\overline{A} \bullet \overline{B}}{\lvert A\rVert B\rvert}$	5	Direction Cosine $\overrightarrow{OP}$ Cos α = $\frac{x}{\lvert\overrightarrow{OP}\rvert}$ Cos β = $\frac{y}{\lvert\overrightarrow{OP}\rvert}$ Cos γ = $\frac{z}{\lvert\overrightarrow{OP}\rvert}$
2	$\overline{A} \times \overline{B} = \begin{pmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$	4	Unit vector $\hat{u} = \frac{\overline{u}}{\lvert u\rvert}$	6	Area of a triangle $\frac{1}{2}\lvert\overrightarrow{AB} \times \overrightarrow{BC}\rvert$

Non Linear Equation (Circle)

1	$(x-a)^2 + (y-b)^2 = r^2$
2	$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad r = \sqrt{g^2 + f^2 - c} \qquad center = (-g,-f)$
3	Equation of a tangent , $y - y_1 = m(x - x_1)$

Parabola

1	Vertical	i. $x^2 = 4ay$	ii. $(x-h)^2 = 4a(y-k)^2$
2	Horizontal	i. $y^2 = 4ax$	ii. $(y-k)^2 = 4a(x-h)^2$
3	Vertex	$v = (h,k)$	
4	Focus	$(h \pm a, k)(horizontal)$	$(h, k \pm a)(vertical)$
5	Directrix	i. $x = h - a$	ii. $y = k - a$

SECTION D: (JKM)

STRUCTURED (25 marks)

INSTRUCTION:

This section consists of **TWO (2)** structured questions. Answer **ONE (1)** question only.

QUESTION 5

a) Find the equation of a circle with its centre (5,-2) and touches the line of $4x - 3y + 4 = 0$. [CLO4:C3] (4 marks)

b) Find the equations of the tangent and normal to the circle $x^2 + y^2 - 3x + 5y - 2 = 0$ at point A(-1,7). [CLO4:C3] (9 marks)

c) Diagram 5(c) shows a parabola. Find the focus, directrix, axis and equation of the parabola. [CLO4:C3] (12 marks)

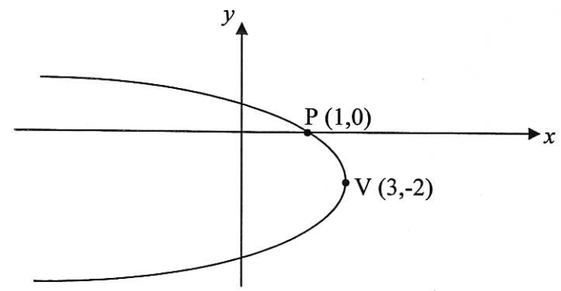

Diagram 5(c)

Laplace Transform

BIL	$f(t)$	$F(s)$	BIL	$f(t)$	$F(s)$
1	a	$\frac{a}{s}$	13	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a^2)+\omega^2}$
2	t^n	$\frac{n!}{s^{n+1}}$	14	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
3	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	15	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$
4	e^{-at}	$\frac{1}{s+a}$	16	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$
5	te^{-at}	$\frac{1}{(s+a)^2}$	17	$e^{at}\sinh\omega t$	$\frac{\omega}{(s^2-a^2)-\omega^2}$
6	$t^n e^{at}, n=1,2,3,...$	$\frac{n!}{(s-a)^{n+1}}$	18	$e^{at}\cosh\omega t$	$\frac{s-a}{(s^2-a^2)-\omega^2}$
7	$t^n.f(t)$	$(-1)^n\frac{d^n}{ds^n}[F(s)]$	19	$f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$
8	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	20	$\int_o^t f(u)du$	$\frac{F(s)}{s}$
9	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	21	$f(t-a)u(t-a)$	$e^{-as}F(s)$
10	$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	22	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s)-y(0)$
11	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	23	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

Trigonometry Identity

1	$\sin 2x = 2\sin x kosx$
2	$kos2x = 2kos^2 x - 1 = 1 - \sin^2 x$

QUESTION 6

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