STRUCTURED (100 marks)

INSTRUCTION:

This section consists of **SIX (6)** structured questions. Answer **FOUR (4)** questions only.

QUESTION 1

a) Calculate the value for each of the following functions :

i. $\sinh(0.82)$ (2 marks)

ii. $\cosh\left(\frac{5}{6}\right)$ (2 marks)

iii. $\sec h\left(\frac{1}{\sqrt{5}}\right)$ (3 marks)

iv. $\cosh^{-1}\left(\frac{3}{2}\right)$ (2 marks)

v. $tanh^{-1}(-0.322)$ (2 marks)

b) Complete the table 1-(b) and sketch the graph of $y = \cosh(2x)$

(8 marks)

x	-2	-1	0	1	2
y					

Table 1-(b)

POLITEKNIK
Jabatan Pengajian Politeknik

EXAMINATION AND EVALUATION DIVISION
DEPARTMENT OF POLYTECHNIC EDUCATION
(MINISTRY OF HIGHER EDUCATION)

MATHEMATICS, SCIENCE AND COMPUTER DEPARTMENT

FINAL EXAMINATION

JUNE 2012 SESSION

B5001: ENGINEERING MATHEMATICS 5

DATE: 19 NOVEMBER 2012 (MONDAY)
DURATION: 2HOURS (2.30PM – 4.30PM)

This paper consists of **SIX (6)** pages including the front page.

Structured (6 questions - answer 4 questions only)

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c) Sketch the quadrant graph and find the principal value for each of the following inverse trigonometric functions:-

i. $cos^{-1}(-\frac{\sqrt{3}}{2})$ (3 marks)

ii. $tan^{-1}(\frac{1}{2})$ (3 marks)

QUESTION 2

a) Differentiate each of the following equation with respect to x:

i. $y = \cos ec^{-1}(e^x)$ (4 marks)

ii. $y = 5\ln(\cosh 4x)$ (3 marks)

iii. $y = \tanh^{-1}\frac{2x}{3}$ (6 marks)

b) Find $\frac{dy}{dx}$ if:

i. $y = e^{2x}\sinh^{-1}x$ (4 marks)

ii. $y = \cot^{-1}4x^3$ (3 marks)

c) Use implicit differentiation to find $\frac{dy}{dx}$ for the following equation:

$5y^2 + \sin y = x^3$ (5 marks)

QUESTION 3

a) Given $z = x\cos 2y - y\sin x^2$, find:

i. $\frac{\partial z}{\partial x}$ (3 marks)

ii. $\frac{\partial z}{\partial y}$ (3 marks)

b) Given $z = 6x^3 - 4xy - 3y^2$, find:

i. $\frac{\partial^2 z}{\partial x^2}$ (2 marks)

ii. $\frac{\partial^2 z}{\partial y^2}$ (2 marks)

iii. $\frac{\partial z}{\partial x\,\partial y}$ (2 marks)

iv. $\frac{\partial z}{\partial y\,\partial x}$ (2 marks)

c) If $u = \ln(3x^2y)$, find $\frac{\partial u}{\partial x}$ *and* $\frac{\partial u}{\partial y}$. (4 marks)

d) The height of a cylinder is 250 cm and increasing at a rate of 0.4 m /s. The radius of its base is 100 cm and decreasing at a rate of 0.5 m/s. Find the rate of change for its volume.

(7 marks)

QUESTION 4

Integrate the following functions with the respect of x:

i. $\int \frac{dx}{\sqrt{25-4x^2}}$ (5 marks)

ii. $\int \frac{dx}{\sqrt{9x^2-3}}$ (5 marks)

iii. $\int 7\tanh(3x-\frac{1}{2})\,dx$ (5 marks)

iv. $\int \frac{e^{2x}}{1+e^{4x}}dx$ (5 marks)

v. $\int \frac{1}{\sqrt{1-4x-x^2}}dx$ (5 marks)

QUESTION 5

a) Integrate the following functions using **Integration by Partial Fractions**:

i. $\int \frac{x}{(x-3)(x+3)}dx$ (5 marks)

ii. $\int \frac{x^3+2}{(x-2)(x+3)}dx$ (8 marks)

b) Solve the following functions using **Integration by Parts Method:**

i. $\int x\cdot\cos\frac{x}{5}dx$ (7 marks)

ii. $\int(6x-5)e^{7x}\,dx$ (5 marks)

QUESTION 6

a) Solve the following differential equations :

i. $x^2(1-y)\frac{dy}{dx}=(1+x)y$ (5 marks)

ii. $x\frac{dy}{dx}+y=x^3$ (8 marks)

b) Solve the following second order differential equation:

$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+9y=0$ (6 marks)

c) Form the differential equation for :

$y = Ax^2 - Bx + x$ (6 marks)

INTEGRATION OF HYPERBOLIC FUNCTIONS

$$\int \sinh x\,dx = \cosh x + c$$

$$\int \cosh x\,dx = \sinh x + c$$

$$\int \sec h^2 x\,dx = \tanh x + c$$

$$\int \cos ech^2 x\,dx = -\coth x + c$$

$$\int \sec h\,x \tanh x\,dx = -\sec h\,x + c$$

$$\int \cos ech\,x \coth x\,dx = -\cos ech\,x + c$$

INTEGRATION OF INVERSE FUNCTIONS

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c$$

$$\int \frac{-du}{\sqrt{a^2-u^2}} = \cos^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$$

$$\int \frac{-du}{a^2+u^2} = \frac{1}{a}\cot^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + c$$

$$\int \frac{-du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\cos ec^{-1}\left|\frac{u}{a}\right| + c$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}\frac{u}{a} + c,$$

$$\int \frac{du}{u^2-a^2} = -\frac{1}{a}\coth^{-1}\frac{u}{a} + c,$$

$$\int \frac{du}{|u|\sqrt{a^2+u^2}} = -\frac{1}{a}\cos ech^{-1}\left|\frac{u}{a}\right| + c$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\sec h^{-1}\frac{u}{a} + c$$

B 5001 ENGINEERING MATHEMATICS FORMULAE

TRIGONOMETRIC IDENTITIES

$$\cos^2 x + \sin^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\cos\sec^2 x = 1 + \cot^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

HYPERBOLIC IDENTITIES

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sec h^2 x = 1 - \tanh^2 x$$

$$\cos ech^2 x = \coth^2 x - 1$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$= 1 + 2\sinh^2 x$$

$$= 2\cosh^2 x - 1$$

$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$

HIPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\cos ech\ x = \frac{2}{e^x - e^{-x}},\ x \neq 0$$

$$\sec h\ x = \frac{2}{e^x + e^{-x}}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}},\ x \neq 0$$

INVERSE HIPERBOLIC FUNCTIONS

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), -\infty < x < \infty$$

$$\cosh^{-1} x = \pm\ln\left(x + \sqrt{x^2-1}\right),\ x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x},\ |x| < 1$$

$$\sec h^{-1} x = \ln\left[\frac{1+\sqrt{1-x^2}}{x}\right],\ 0 < x \leq 1$$

$$\cos ech^{-1} x = \ln\left[\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right],\ x \neq 0$$

$$\coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1},\ |x| > 1$$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\cos ec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u\frac{du}{dx}$$

$$\frac{d}{dx}(\cos ec\ u) = -\cos ec\ u\ \cot u\frac{du}{dx}$$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS

$$\frac{d}{dx}(\sinh u) = \cosh u\frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u) = \sinh u\frac{du}{dx}$$

$$\frac{d}{dx}(\tanh u) = \sec h^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\coth u) = -\cos ech^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\sec h\,u) = -\sec h\,u \tanh u\frac{du}{dx}$$

$$\frac{d}{dx}(\cos ech\,u) = -\cos ech\,u\ \coth u\frac{du}{dx}$$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS

$$\frac{d}{dx}\left(\sinh^{-1} u\right) = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\tanh^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\coth^{-1} u\right) = \frac{-1}{u^2-1}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\sec h^{-1} u\right) = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos ech^{-1} u\right) = \frac{-1}{|u|\sqrt{1+u^2}}\frac{du}{dx}$$

DIFFERENTIATION OF INVERSE TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos^{-1} u\right) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cot^{-1} u\right) = \frac{-1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos ec^{-1} u\right) = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$