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B4001: ENGINEERING MATHEMATICS 4

STRUCTURED (100 MARKS)

INSTRUCTION:

This section consists of EIGHT (8) structured questions. Answer FOUR (4) questions only.

For JKE & JKP students: Answer Four (4) questions from question 1, 2, 3, 4, 5 or 6 For JKM students: Answer Four (4) questions from question 1, 2, 3, 4, 7 or 8

QUESTION 1

- a) Find, in ascending power of x the first four terms in the expansion of:
 - i) $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$

(4 marks)

ii) $(4+4x)^{-2}$

(6 marks)

b) Expand $(1-x)^{30}$ in ascending power of x up to the first four terms. Then, find the value of $(0.973)^{30}$ correct to five decimal places.

(9 marks)

Using the Binomial Theorem, find the value up to five decimal places for $\frac{1}{(0.995)^3}$

(6 marks)

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EXAMINATION AND EVALUATION DIVISION DEPARTMENT OF POLYTECHNIC EDUCATION (MINISTRY OF HIGHER EDUCATION)

MATHEMATICS, SCIENCE & COMPUTER DEPARTMENT

FINAL EXAMINATION
JUNE 2012 SESSION

B4001: ENGINEERING MATHEMATICS 4

DATE: 17 NOVEMBER 2012 (SATURDAY) DURATION: 2 HOURS (8:30 AM – 10:30 AM)

This paper consists of EIGHT (8) pages including the front page

Structured questions (8 questions – answer only 4 questions)
For JKE & JKP students: Answer Four (4) questions from question 1, 2, 3, 4, 5
or 6

For JKM students: Answer Four (4) questions from question 1, 2, 3, 4, 7 or 8

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THE CHIEF INVIGILATOR

QUESTION 3

- a) If $\tilde{p} = 5\tilde{i} 2\tilde{j} + 7\tilde{k}$ and $\tilde{q} = 2\tilde{i} 7\tilde{j} 6\tilde{k}$, find:
 - i) $4\widetilde{p} \times 2\widetilde{q}$

(5 marks)

ii) $(3\widetilde{p} \bullet 6\widetilde{q})\widetilde{p}$

(5 marks)

- b) Given vector $\overrightarrow{OA} = 3\tilde{i} 3\tilde{j} + 2\tilde{k}$, $\overrightarrow{OB} = 4\tilde{i} + 3\tilde{j} 5\tilde{k}$ and $\overrightarrow{OC} = -5\tilde{i} + \tilde{j} 3\tilde{k}$ determine:
 - i) \overrightarrow{AB}

(2 marks)

ii) \overrightarrow{BC}

(2 marks)

iii) $\overrightarrow{AB} \times \overrightarrow{BC}$

(3 marks)

c) Find the angle between vector $4\tilde{i} + \tilde{j} - 3\tilde{k}$ and vector $2\tilde{i} - 3\tilde{j} + 3\tilde{k}$

(8 marks)

QUESTION 2

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a) Find the first four terms for the expansion of the following function:

$$(x+2x^2)e^{-2x}$$

(6 marks)

ii)
$$ln[(1+x)^3(1-2x)]$$

(7 marks)

Find the coefficient of x^4 for the expansion of $e^{\frac{x}{4}}$.

(4 marks)

c) Find the Taylor Series of $f(x) = \cos(1+3x)$ at $x_0 = \frac{\pi}{3}$ until the term with x^3 .

(8 marks)

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QUESTION 5

- a) Find the Laplace Transform for:
 - i) $f(t) = \cos 8t \sin 8t$

(3 marks)

ii) $f(t) = 4e^{3t} \sin 5t$

(3 marks)

iii) $f(t) = t^7 - 6t^5 - 4t^3$

(4 marks)

 $iv) f(t) = -4e^{-5t} \sinh t$

(4 marks)

b) Use the definition method to find the Laplace Transform for the following functions:

i) f(t) = -7

(5 marks)

ii) $f(t) = e^{-6t}$

(6 marks)

QUESTION 4

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Express the following fractions into partial fractions:

$$a) \qquad \frac{3}{(1-x)^3}$$

(5 marks)

b)
$$\frac{x^2-1}{x^2(2x+1)}$$

(6 marks)

c)
$$\frac{x^3}{(x+1)(x-3)}$$

(7 marks)

d)
$$\frac{2x^3 + x^2 - 15x - 5}{(x+3)(x-2)}$$

(7 marks)

QUESTION 7

a) Sketch the graph of the exponential function, $y = 3e^{2x}$ for $-2 \le x \le 2$.

(8 marks)

b) Find the centre point and the radius of a circle $2x^2 + 2y^2 - 4x + 6y + 3 = 0$.

(7 marks)

Find the equations of the tangent and normal at the point (2,-3) to the circle $3x^2 + 3y^2 + 5x - 4y - 1 = 0.$

(10 marks)

QUESTION 8

- a) Find the equation of each parabola based on the following conditions:
 - i. Vertex at (0, 0) and focus at (0, 6).

(5 marks)

ii. Focus at (6, 2) and directrix at x = 2.

(7 marks)

b) Find the center, major vertex and minor vertex of an ellipse $x^2 + 4y^2 - 8x - 32y = 20$.

(13 marks)

QUESTION 6

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a) Find the Inverse Laplace Transform for the following expressions:

i.
$$F(s) = \left(\frac{5}{s+2}\right) + \left(\frac{5}{s^2 - 25}\right)$$

(2 marks)

ii.
$$F(s) = \frac{3}{(4s^2 - 16)}$$

(3 marks)

iii.
$$F(s) = \frac{7}{\left(s-4\right)^3}$$

(4 marks)

b) Find the Inverse Laplace Transform for the following expressions using the Partial Fraction Method.

i.
$$\frac{4s+4}{s(s-4)}$$

(7 marks)

ii.
$$\frac{7s^2-3}{(s+2)^2(s-4)}$$

(9 marks)

Parabola

1.	Vertical	i. $x^2 = 4ay$	ii. $(x-h)^2 = 4a(y-k)$
2.	Horizontal	i. $y^2 = 4ax$	ii. $(y-k)^2 = 4a(x-h)$
3.	Vertex	v = (h, k)	,
4.	Focus	(h+a,k) – horizontal	(h, k+a) – vertical
5.	Directrix	i. $x = h - a$	ii. $y = k - a$

Ellipse

Hyperbola

1.	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	1. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ horizontal	
		$2. \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \qquad \text{vertical}$	

Laplace Transform

NUM	f(t)	F(s)	9	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
1	а	$\frac{a}{s}$	10	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
2	at	$\frac{a}{s^2}$	11	sinh <i>ωt</i>	$\frac{\omega}{(s^2-\omega^2)}$
3	e^{-at}	$\frac{1}{s+a}$	12	cosh ωt	$\frac{s}{(s^2-\omega^2)}$
4	te ^{-at}	$\frac{1}{(s+a)^2}$	13	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
5	t ⁿ	$\frac{n!}{s^{n+1}}$	14	$\frac{df}{dt}$	sF(s)-f(0)
6	sin <i>oot</i>	$\frac{\omega}{(s^2+\omega^2)}$	15	$\int_{o}^{t} f(u) du$	$\frac{F(s)}{s}$
7	cos wt	$\frac{s}{(s^2+\omega^2)}$	16	f(t-a)u(t-a)	$e^{-as}F(s)$
8	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	17	$t^n.f(t)$	$(-1)^n \frac{d^n}{ds^n} [F(s)]$
		89-1			

Trigonometric Identities

1	$\sin 2x = 2\sin x \cos x$	
2	$\cos 2x = 2\cos^2 x - 1 = 1 - \sin^2 x$	

FORMULA OF ENGINEERING MATHEMATICS 4 (B4001)

Binomial Expansion

1.	$(a+x)^n = a^n + {}^nC_1 a^{n-1} x + {}^nC_2 a^{n-2} x^2 + \dots + x^n$	(n = positive integer)
2.	$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots \infty$	(n = negative interger or fraction)

Power Series

101	ter beries
1.	$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \frac{x^{4}}{4!} + \dots + \frac{x^{n}}{n!}$
2.	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n}$
3.	$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots + \frac{f''(0)x^n}{n!}$ (MACLAURIN)
4.	$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0)^2}{2!} + \frac{f'''(x_0)(x - x_0)^3}{3!} + \dots + \frac{f''(x_0)(x - x_0)^n}{n!} $ (TAYLOR)

Vector and Scalar

1.	$\overline{A} \bullet \overline{B} = a_1 a_2 + b_1 b_2 + c_1 c_2$	3.	$\cos \theta = \frac{\overline{A} \bullet \overline{B}}{ A B }$	5.	Direction Cosine \overrightarrow{OP} $\cos \alpha = \frac{x}{\left \overrightarrow{OP}\right }$ $\cos \beta = \frac{y}{\left \overrightarrow{OP}\right }$ $\cos \gamma = \frac{z}{\left \overrightarrow{OP}\right }$
2.	$\overline{A} \times \overline{B} = \begin{pmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$	4.	Unit vector $\hat{u} = \frac{\overline{u}}{ u }$	6.	Area of a triangle $\frac{1}{2} \left \overrightarrow{AB} \times \overrightarrow{BC} \right $

Non Linear Equation (Circle)

1.	$(x-a)^2 + (y-b)^2 = r^2$		
2.	$x^2 + y^2 + 2gx + 2fy + c = 0$	$r = \sqrt{g^2 + f^2 - c}$	center = (-g, -f)
3.	Equation of a tangent, $y - y_1 =$	$m(x-x_1)$	