STRUCTURED

INSTRUCTION:

This section consists of **SIX (6)** structured questions.

Answer **FOUR (4)** questions only.

QUESTION 1

(a) Expand each of the following expressions using the Binomial Theorem to the first four terms :

i) $(2-3x^2)^5$

(4 marks)

ii) $\left(1+\frac{x}{6}\right)^{-\frac{2}{5}}$

(4 marks)

(b) Find the coefficient of x^5 for $\left(x^2+\frac{2}{3x}\right)^{10}$.

(8 marks)

(c) Expand $\sqrt[3]{1-x}$ to the first four terms. Then, find the value of $\sqrt[3]{0.998}$ correct to five decimal places.

(9 marks)

POLITEKNIK
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EXAMINATION AND EVALUATION DIVISION
DEPARTMENT OF POLYTECHNIC EDUCATION
(MINISTRY OF HIGHER EDUCATION)

MATHEMATICS, SCIENCE & COMPUTER DEPARTMENT

FINAL EXAMINATION

DECEMBER 2011 SESSION

B4001 : ENGINEERING MATHEMATICS 4

DATE : 23 APRIL 2012 (MONDAY)
DURATION : 2 HOURS (2.30 PM – 4.30 PM)

This paper consists of **ELEVEN (11)** pages including the front page and appendix.

This paper consists of **EIGHT(8)** questions.
Answer **FOUR (4)** questions only.
For **JKE & JKP** students : Answer **FOUR (4)** questions from question **1, 2, 3, 4, 5 or 6**
For **JKM** students : Answer **FOUR (4)** questions from question **1, 2, 3, 4, 7 or 8**

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QUESTION 3

(a) Given vector $\overrightarrow{OA} = 12\tilde{i} - 43\tilde{j} - 8\tilde{k}$ and $\overrightarrow{OB} = 2\tilde{i} + 6\tilde{j} + 10\tilde{k}$, determine :

i) $\overrightarrow{OA} \cdot \overrightarrow{OB}$

(2 marks)

ii) $\overrightarrow{OA}$ x $\overrightarrow{OB}$

(4 marks)

iii) the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$

(6 marks)

(b) If position vectors $\vec{K}$, $\vec{L}$ and $\vec{M}$ are defined by $\vec{K} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$, $\vec{L} = 3\tilde{i} + 2\tilde{j} - 4\tilde{k}$ and $\vec{M} = -\tilde{i} + 3\tilde{j} - 2\tilde{k}$, determine :

i) vector $\overrightarrow{KL}$

(4 marks)

ii) vector $\overrightarrow{LM}$

(4 marks)

iii) $\overrightarrow{KL} \cdot 2\overrightarrow{M}$

(5 marks)

QUESTION 2

(a) Find the first four terms of the following functions :

i) $\ln(-2x^2 - x + 1)$

(5 marks)

ii) $(1 - x)\ln(1 - x)^2$

(5 marks)

(b) Find the first four terms in the expansion of $(x+1)e^x$. Then, find the coefficient for x^3 .

(5 marks)

(c) Find the first four terms of the Taylor Series for $f(x) = \dfrac{1}{x+1}$ when $x_0 = 1$.

(10 marks)

QUESTION 5

(a) Find the Laplace Transform for the following functions using the definition below:

$$F(s) = \int_0^{\infty} f(t)e^{-st}$$

i) $f(t) = m$

(5 marks)

ii) $f(t) = e^{mt}$

(7 marks)

(b) Find the Laplace Transform for the following functions by using the Laplace Transform Table.

i) $f(t) = \cos 5t + \sin 5t$

(2 marks)

ii) $f(t) = 2e^{2t}\sin 6t$

(5 marks)

iii) $f(t) = t^3e^{3t} + t^2(t-1)^2$

(6 marks)

QUESTION 4

Express the following in the form of partial fractions.

(a) $\dfrac{2x-4}{(1-2x)(1+x)}$

(5 marks)

(b) $\dfrac{x^2+1}{(x+3)^3}$

(6 marks)

(c) $\dfrac{2x-3}{(x-2)\ (x^2+3x+1)}$

(8 marks)

(d) $\dfrac{3x^2-4}{x(x^2+1)}$

(6 marks)

QUESTION 7

a) Find the equation of the circle centered at point (2, -5) with a radius of 3.

(5 marks)

b) Find the radius and centre point of a circle using the following equation.

$4x^2 + 4y^2 - 16x - 24y + 51 = 0$

(10 marks)

c) Determine the equation of the circle that passes through the intersection of these two circles: $x^2 + y^2 + 7x + 2y - 7 = 0$ and $x^2 + y^2 + 3x - 2y - 1 = 0$ and through the point (1, 2).

(10 marks)

QUESTION 6

a) Determine the Inverse Laplace Transform of:

i. $F(s) = \frac{10}{s^3} + \frac{5}{(s-3)}$

(3 marks)

ii. $F(s) = \frac{1}{s^2 + 16}$

(3 marks)

iii. $F(s) = \frac{s+2}{s^2 + 5}$

(4 marks)

b) Find the Inverse Laplace Transform for the following expressions using the Partial Fraction Method.

i. $\frac{11}{(s^2 - 16)}$

(6 marks)

ii. $\frac{7s - 2}{(s+4)(s^2 + 9)}$

(9 marks)

QUESTION 8

(a) Given the parabola $(y+2)^2 = \frac{1}{2}(x-3)$,

i) find the vertex

(4 marks)

ii) find the focus

(2 marks)

iii) find the diretrix equation

(2 marks)

iv) find the parabolic axis

(2 marks)

v) sketch the graph of the parabola

(5 marks)

(c) Sketch the ellipse for the given equation $\frac{x^2}{3} + \frac{y^2}{5} = 1$

(10 marks)

Parabola

1.	Vertical	i. $x^2 = 4ay$	ii. $(x-h)^2 = 4a(y-k)$
2.	Horizontal	i. $y^2 = 4ax$	ii. $(y-k)^2 = 4a(x-h)$
3.	Vertex	$v = (h,k)$	
4.	Focus	$(h+a,k) - horizontal$	$(h,k+a) - vertical$
5.	Directrix	i. $x = h-a$	ii. $y = k-a$

Ellipse

1.	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hyperbola

1.	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	horizontal
2.	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	vertical

Laplace Transform

NUM	$f(t)$	$F(s)$	9	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
1	a	$\frac{a}{s}$	10	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
2	at	$\frac{a}{s^2}$	11	$\sinh\omega t$	$\frac{\omega}{(s^2-\omega^2)}$
3	e^{-at}	$\frac{1}{s+a}$	12	$\cosh\omega t$	$\frac{s}{(s^2-\omega^2)}$
4	te^{-at}	$\frac{1}{(s+a)^2}$	13	$f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$
5	t^n	$\frac{n!}{s^{n+1}}$	14	$\frac{df}{dt}$	$sF(s)-f(0)$
6	$\sin\omega t$	$\frac{\omega}{(s^2+\omega^2)}$	15	$\int_0^t f(u)du$	$\frac{F(s)}{s}$
7	$\cos\omega t$	$\frac{s}{(s^2+\omega^2)}$	16	$f(t-a)u(t-a)$	$e^{-as}F(s)$
8	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	17	$t^n.f(t)$	$(-1)^n\frac{d^n}{ds^n}[F(s)]$

Trigonometric Identities

1	$\sin 2x = 2\sin x\cos x$
2	$\cos 2x = 2\cos^2 x - 1 = 1 - \sin^2 x$

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Binomial Expansion

1.	$(a+x)^n = a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \ldots\ldots + x^n$	(n = positive integer)
2.	$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \ldots\infty$	(n = negative interger or fraction)

Power Series

1.	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots. + \frac{x^n}{n!}$	
2.	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots. + (-1)^{n-1}\frac{x^n}{n}$	
3.	$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots.. + \frac{f^n(0)x^n}{n!}$	(MACLAURIN)
4.	$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)(x-x_0)^3}{3!} + \ldots. + \frac{f^n(x_0)(x-x_0)^n}{n!}$	(TAYLOR)

Vector and Scalar

1.	$\overline{A}\bullet\overline{B} = a_1a_2 + b_1b_2 + c_1c_2$	3.	$\cos\theta = \frac{\overline{A}\bullet\overline{B}}{\lvert A\rvert\lvert B\rvert}$	5.	Direction Cosine $\overrightarrow{OP}$ cos α = $\frac{x}{\lvert\overrightarrow{OP}\rvert}$ cos β = $\frac{y}{\lvert\overrightarrow{OP}\rvert}$ cos γ = $\frac{z}{\lvert\overrightarrow{OP}\rvert}$
2.	$\overline{A}\times\overline{B} = \begin{pmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$	4.	Unit vector $\hat{u} = \frac{\overline{u}}{\lvert u\rvert}$	6.	Area of a triangle $\frac{1}{2}\lvert\overrightarrow{AB}\times\overrightarrow{BC}\rvert$

Non Linear Equation (Circle)

1.	$(x-a)^2 + (y-b)^2 = r^2$		
2.	$x^2 + y^2 + 2gx + 2fy + c = 0$	$r = \sqrt{g^2 + f^2 - c}$	$center = (-g,-f)$
3.	Equation of a tangent , $y - y_1 = m(x - x_1)$		